2018 年度　確率数理要論　レポート２
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- 提出方法：工学部 6 号館 1 階にある鈴木のポストに紙媒体で提出すること.
- 学生証番号・氏名を明記すること.
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Problem 1 For a Markov chain, suppose that a state x is recurrent and a state y is accessible from x ($x \to y$). Then, show that x and y communicate with each other ($x \leftrightarrow y$) and y is also recurrent.

Problem 2 Suppose that independent random variables X and Y obey Poisson distributions with means λ_X and λ_Y respectively. Then, derive the distribution of $X+Y$.

Problem 3 Let $(B_t)_{t\geq 0}$ be a Brownian motion and $f:\mathbb{R}\to\mathbb{R}$ be a twice continuously differentiable function. Then, show that

$$X_t = f(B_t) - \frac{1}{2}\int_0^t f''(B_s)\mathrm{d}s$$

is a martingale (Hint: use Ito's formula).

Problem 4 (Brownian bridge) Solve the following stochastic differential equation:

$$\mathrm{d}X_t = \frac{1-X_t}{1-t}\mathrm{d}t + \mathrm{d}B_t \quad (X_0 = 0,\ 0 \leq t < 1).$$